

Infrared dynamics of the massive ϕ^4 theory on de Sitter space

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Abstract

We study massive real scalar ϕ^4 theory in the expanding Poincare patch of de Sitter space. We calculate the leading two loop infrared contribution to the two-point function in this theory. We do that for the massive fields both from the principal and complementary series. As can be expected at this order light fields from the complementary series show stronger infrared effects than the heavy fields from the principal one. For the principal series, unlike the complementary one, we can derive the kinetic equation from the system of Dyson-Schwinger equation, which allows us to sum up the leading infrared contributions from all loops. We find two peculiar solution of the kinetic equation. One of them describes stationary Gibbons-Hawking type distribution for the density per comoving volume. Another solution shows explosive (square root of the pole in finite proper time) growth of the particle number density per comoving volume. That signals the possibility of the destruction of the expanding Poincare patch even by the very massive fields. We conclude with the consideration of the infrared divergences in global de Sitter space and in its contracting Poincare patch.

1 Introduction

There are large infrared (IR) loop contributions even in the massive field theories in the Poincare patch (PP) of de Sitter (dS) space [1], [2], [3] (see also [4]–[8] for the massless fields). In global dS space there are IR divergences [1], [9], [10] (see also [11]), which lead to the inevitable breaking of the dS isometry in the loops for any initial state. They are specific to dS space physics and are absent e.g. in AdS space [12].

For the alternative point of view on the IR properties of the massive field theories in dS space see [13], [14]. However, these papers heavily rely on the analytical properties of the correlators as functions of the dS invariant distances between their arguments. But such an approach does not work when the dS isometry is broken by the initial density perturbation (or by the IR divergence in global dS). In fact, then the correlation functions stop to depend only on the invariant distance and become functions of each point separately. The motivation to

include initial density perturbation is that it is rather unphysical to address the question of the stability of the system under such conditions when all its symmetries are respected exactly. We propose to consider excitations above the highly symmetric state and to trace where do they evolve in the future infinity — to the symmetric state or not? In global dS one even does not have to consider initial density perturbations, because dS isometry is broken in the loops by the IR divergences.

In [1], [2], [3], [9], [10], [15] the large IR contributions to the two point functions were calculated for the massive real scalar field theory with the cubic, ϕ^3 , selfinteraction. The cubic potential has the runaway instability. Moreover, one-loop higher power correlation functions of the ϕ^3 theory may potentially show strong IR effects, which are absent in the ϕ^4 theory at least in 4D. To show that those IR effects, which are observed in our previous papers, are universal rather than due to do the specific properties of the ϕ^3 theory, we consider here the scalar field theory with ϕ^4 selfinteraction. Furthermore, in the pervious papers only massive fields from the principal series have been considered. Here we extend those considerations to the complementary series.

We calculate loop corrections to the so called Keldysh propagator in PP of dS space. Unlike ϕ^3 case, in the ϕ^4 theory there are no any large IR contributions in the first loop. However, at the two loop order we find such contributions in the sunset diagrams. In the case of the fields from the principal series the contribution is linear logarithmic in the physical momentum. For the complementary series the situation is more subtle and depends on the type of α -harmonics, but always is power like, i.e. stronger than for the principal series.

We are not yet capable to perform the summation of the higher loop contributions for the case of the complementary series. But for the principal series it is possible to do such a summation. That is done via a suitable IR ansatz for the solution of the system of Dyson-Schwinger (DS) equations, which allows to reduce it to a generalization of the Boltzmann's kinetic equation. The latter one has a clear physical meaning and describes various particle decay and creation processes in the dS space (see also [17]–[20], [11]). We solve this kinetic equation in two cases. One corresponds to the very mild initial density perturbation over the initial Bunch-Davies state. In such a case the state of the theory relaxes to the eventual Gibbons-Hawking type stationary distribution for the number density per comoving volume. Another situation corresponds to the strong enough initial density perturbation, which, however, is still much smaller than the cosmological constant. In this case the state of the theory shows the explosive (square root of the pole in finite proper time) growth of the particle number density per comoving volume.

We conclude with the consideration of the contracting PP of dS space and of the global dS and draw similar conclusions to those which have been made in [10] for the ϕ^3 theory.

2 The setup of the problem

D -dimensional de Sitter (dS) space-time is the hyperboloid, $X_0^2 - X_i^2 = -1$, $i = 1, \dots, D$, inside $(D + 1)$ -dimensional Minkowski space-time, $ds^2 = dX_0^2 - dX_i^2$. Throughout this paper we set its curvature to 1 and mostly consider its half (e.g. $X_0 \geq X_D$), which is referred to as expanding Poincare patch (PP): $ds^2 = \frac{1}{\eta^2}(d\eta^2 - d\vec{x}^2)$, where $\eta \in (+\infty, 0)$. Note that while $\eta \rightarrow +\infty$ is the past, $\eta \rightarrow 0$ corresponds to the future infinity. The action of the theory which we are going to study is

$$S = \int d^D x \sqrt{|g|} \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - m^2 \phi^2 - \frac{\lambda}{4!} \phi^4 \right]. \quad (1)$$

Throughout this paper we always assume that $m > 0$.

Corresponding free harmonics in the Fourier expansion $\phi(x, \eta) = \int d^{D-1} k \eta^{\frac{D-1}{2}} [a_k h(k\eta) e^{-ikx} + a_k^\dagger h^*(k\eta) e^{ikx}]$, $k = |\vec{k}|$ are defined via $h(x)$, which is a solution of the Bessel equation with the index $i\mu = i\sqrt{m^2 - \left(\frac{D-1}{2}\right)^2}$. The choice of such a solution specifies the dS invariant vacuum $a_k |vac\rangle = 0$. E.g. Bunch-Davies (BD) state (or in-vacuum in PP) corresponds to $h(p\eta) = \frac{\sqrt{\pi}}{2} e^{-\frac{\pi\mu}{2}} H_{i\mu}^{(1)}(p\eta)$, where $H_{i\mu}^{(1)}$ is the Hankel function of the first kind. All other harmonics and vacua can be obtained from those of BD via a one parameter family of the Bogolyubov rotations [21], [22]. For example, the out-vacuum corresponds to the so called out-Jost-harmonics, $h(p\eta) = \sqrt{\frac{\pi}{\sinh(\pi\mu)}} J_{i\mu}(p\eta)$, where $J_{i\mu}$ is the Bessel function of the first kind. Conjugate harmonics in the latter case are given by the Y 's — Bessel functions of the second kind.

In the nonstationary situation every particle is described by the matrix propagator (see e.g. [23] for the Feynman rules in the ϕ^3 scalar field theory on dS space) whose entries are — the Keldysh propagator, $G_K = \frac{1}{2} \langle \{\phi(x), \phi(y)\} \rangle$, and the retarded and advanced propagators*, $G_R^A = \mp \langle [\phi(x), \phi(y)] \rangle \theta(\mp \Delta x_0)$. The more detailed discussion of the physics in dS space, which is relevant for the present paper, can be found in [1], [3], [9], [10], [15]. It is instructive for the further discussion to keep in mind that the result of the calculation with the use of the non-stationary Keldysh-Schwinger diagrammatic technique provides a solution to a sort of the Cauchy problem.

Because PP is spatially homogeneous and because below we concentrate on the spatially homogeneous states, we find it convenient to make the spatial Fourier transform of the aforementioned propagators:

$$\begin{aligned} D^{K,R,A}(\eta_1, \eta_2, p) &= \int d^{D-1} x e^{i\vec{p}\vec{x}} G^{K,R,A}(\eta_1, \vec{x}, \eta_2, 0), \\ D^K(\eta_1, \eta_2, p) &= (\eta_1 \eta_2)^{\frac{D-1}{2}} d^K(p\eta_1, p\eta_2), \\ D_R^A(\eta_1, \eta_2, p) &= \mp \theta(\pm \Delta\eta) (\eta_1 \eta_2)^{\frac{D-1}{2}} d^\mp(p\eta_1, p\eta_2), \end{aligned} \quad (2)$$

where $\Delta\eta = \eta_1 - \eta_2$, $p = |\vec{p}|$ and

$$\begin{aligned} d^-(p\eta_1, p\eta_2) &= 2\text{Im} [h(p\eta_1) h^*(p\eta_2)], \\ d^K(p\eta_1, p\eta_2) &= h(p\eta_1) h^*(p\eta_2) \left(\frac{1}{2} + n_p \right) + h(p\eta_1) h(p\eta_2) k_p + \text{c.c.} \end{aligned} \quad (3)$$

Here we use the convenient notations $n_p = \langle a_p^\dagger a_p \rangle$ and $\kappa_p = \langle a_p a_{-p} \rangle$. In the non-stationary situation n_p and k_p can be zero only on the tree-level, i.e. if the initial state is chosen to be $|vac\rangle$.

It is worth stressing here that in our paper we always study two-point correlation function. According to (2)-(3), the quantities $n_p = \langle a_p^\dagger a_p \rangle$ and $k_p = \langle a_p a_{-p} \rangle$ are juts elements of this

*Here $\Delta x_0 = x_0 - y_0$, $\{, \}$ is the anti-commutator and $[,]$ is the commutator; $\theta(\eta)$ is the Heaviside θ -function.

correlation function. Hence, all our conclusions about their behavior have invariant physical meaning. Furthermore, in the situation when the anomalous quantum average k_p vanishes n_p acquires the clear physical meaning — it becomes the particle number density per comoving volume in the given state of the theory. In fact, then free Hamiltonian of the theory acquires the diagonal form. Moreover, then all elements of the kinetic equation which is presented below acquire the clear physical meaning.

Let us provide here a few more arguments favoring our interpretation of n_p as the particle density. Various observers may indeed detect different particle fluxes. However, one should separate Unruh effect from what we would like to call as the real particle production. In Minkowski space both inertial and non-inertial observers see the same state - Minkowski (Poincare invariant) vacuum. However, while inertial observer sees it as the empty space, the non-inertial one sees it as the thermal state. That is due to the specific correlation of the vacuum fluctuations along its world-line [16]. Note that there is no any non-trivial gravitational field in the circumstances under consideration, because Riemannian tensor is exactly zero.

The real particle creation is due to the change of the “vacuum” state under the influence of the background field. That is exactly what happens in the background electric field, in dS space and in the collapsing black hole background.

Rephrasing, we would like to say here that while in Minkowski space there is one type of observers which does not see any particle flux, in dS space there is no such an observer which sees nothing. On general grounds we expect that the least particle flux is seen by inertial observers — they do not see extra Unruh type of flux, so to say. Our calculations are actually done for the inertial observers, because the change from the proper time t to the conformal one, $\eta = e^{-t}$, is just the change of the clock’s rate rather than a transition to some non-trivial motion. In any case all our formulas can be trivially rewritten in the proper time t .

Finally, the IR dynamics depends on the value of the mass m and on the choice of $h(x)$. Below we separately consider the following two cases: $(\frac{D-1}{2})^2 < m^2$ — the principal series, and $(\frac{D-1}{2})^2 > m^2$ — the complementary series. The crucial physical difference between these two cases is due to the fact that while the harmonics of the principal series oscillate at the future infinity, $h(x) \approx A_+ x^{i\mu} + A_- x^{-i\mu}$, $x \rightarrow 0$, those of the complementary series don’t do that, because in the latter case μ is pure imaginary.

3 Two-loop contribution

In this section we calculate loop corrections to the Keldysh propagator, $D^K(\eta_1, \eta_2, p)$, with the initial dS invariant vacuum state at the past infinity of PP. In these settings the tree-level $D^K(\eta_1, \eta_2, p)$ is given by (3) with $n_p = 0$ and $\kappa_p = 0$. It is straightforward to show that the ϕ^4 theory, unlike ϕ^3 one, does not poses any large IR contributions to any propagator at the first loop order ($\sim \lambda$). However, in the second loop order ($\sim \lambda^2$) there is a large IR contribution to D^K , which is of interest for us. We consider the IR limit in which $p\sqrt{\eta_1\eta_2} \rightarrow 0$ and $\eta_1/\eta_2 = \text{const}$ [3], [15], [10]. It corresponds to the situation when both time arguments of the propagator are taken to the future infinity, while the time distance between them is kept finite.

The reason why we pay attention only to the Keldysh propagator is that it defines the state of the theory, i.e. shows the dependence of n_p and k_p on time. Moreover, in the case of principal series it is the only propagator which receives large corrections of the order $\lambda^2 \log(p\sqrt{\eta_1\eta_2}/\mu)$ in

the IR limit in question. In fact, in [2] it was shown that the retarded and advanced propagators of the ϕ^3 theory receive only finite corrections from the first loop — of the order $\lambda^2 \log(\eta_1/\eta_2)$. In the case of the ϕ^4 theory the situation is similar in the second loop. Furthermore it is straightforward to show that for the principal series the large IR contributions to the interaction vertex are also suppressed by higher powers of λ . For the complementary series the situation is more subtle, but we still would like to start their study with the consideration of the loop corrections to D^K .

The large IR loop contribution to D^K (if there is one) at the order λ^2 can be expressed in the form (2), (3) with

$$\begin{aligned} n_p(\eta) &\approx -\frac{\lambda^2}{3(2\pi)^{2(D-1)}} \int d^{D-1}q_1 d^{D-1}q_2 d^{D-1}q_3 \int_{\infty}^{\eta} d\eta_3 \int_{\infty}^{\eta} d\eta_4 (\eta_3\eta_4)^{D-2} \\ &\delta^{(D-1)}(\vec{p} + \vec{q}_1 + \vec{q}_2 + \vec{q}_3) h(p\eta_3)h(q_1\eta_3)h(q_2\eta_3)h(q_3\eta_3)h^*(p\eta_4)h^*(q_1\eta_4)h^*(q_2\eta_4)h^*(q_3\eta_4), \\ k_p(\eta) &\approx \frac{2\lambda^2}{3(2\pi)^{2(D-1)}} \int d^{D-1}q_1 d^{D-1}q_2 d^{D-1}q_3 \int_{\infty}^{\eta} d\eta_3 \int_{\infty}^{\eta_3} d\eta_4 (\eta_3\eta_4)^{D-2} \\ &\delta^{(D-1)}(\vec{p} + \vec{q}_1 + \vec{q}_2 + \vec{q}_3) h^*(p\eta_3)h(q_1\eta_3)h(q_2\eta_3)h(q_3\eta_3)h^*(p\eta_4)h^*(q_1\eta_4)h^*(q_2\eta_4)h^*(q_3\eta_4). \end{aligned} \quad (4)$$

Here $p = |\vec{p}|$, $q_{1,2,3} = |\vec{q}_{1,2,3}|$ and $n_p(\eta)$ is real. Note that if one will take the flat space limit of these expressions, i.e. substitute η by t , $\sqrt{|g|}$ by 1 and $\eta^{(D-1)/2} h(k\eta)$ by $e^{i\epsilon(k)t}$, then n_p and k_p would vanish, when $\eta \rightarrow 0$, as the consequence of the energy conservation.

In deriving these expressions we have used that $h(p\eta)$ depends only on $|\vec{p}|$ and, hence, we can safely change $\vec{p} \rightarrow -\vec{p}$. Also in the IR limit in question inside the leading loop corrections one can neglect the difference between η_1 , η_2 and $\eta = \sqrt{\eta_1\eta_2}$. In such an approximation we drop the subleading, $\lambda^2 \log(\eta_1/\eta_2)$, contributions from the expressions for n_p and k_p . The derivation of (4) is similar to the one performed in [1], [2], [3], [10], [15] for the ϕ^3 theory.

3.1 Principal series

We start with the case $\frac{D-1}{2} < m$ and make the following change of the integration variables in (4): \vec{q}_i to $\vec{l}_i = \vec{q}_i\eta_3$ and η_4 to $v = \frac{\eta_3}{\eta_4}$, $i = 1, 2, 3$. Then we expand $h(p\eta_{3,4}) \approx A_+(p\eta_{3,4})^{i\mu} + A_-(p\eta_{3,4})^{-i\mu}$ as $p\eta_{3,4} \rightarrow 0$ under the integrals, where A_{\pm} are some mass dependent complex constants. After that we neglect[†] p in comparison with $q_{1,2,3}$ on the RHS of (4) and perform the integration over η_3 . The largest IR contributions come from the integrals of the type $\int_{\mu/p}^{\eta} \frac{d\eta_3}{\eta_3} h(p\eta_3) h^*(p\eta_3 v)$ and $\int_{\mu/p}^{\eta} \frac{d\eta_3}{\eta_3} h^*(p\eta_3) h^*(p\eta_3 v)$, where h 's are Taylor expanded. The result is as follows:

$$\begin{aligned} n_{p\eta} &\approx -\frac{\lambda^2 \log(p\eta/\mu)}{3(2\pi)^{2(D-1)}} \int_{\infty}^0 dv v^{D-2} \int d^{D-1}l_1 d^{D-1}l_2 d^{D-1}l_3 \delta^{(D-1)}(\vec{l}_1 + \vec{l}_2 + \vec{l}_3) \times \\ &\times h^*(l_1)h(l_1v)h^*(l_2)h(l_2v)h^*(l_3)h(l_3v) [|A_+|^2 v^{-i\mu} + |A_-|^2 v^{+i\mu}], \\ k_{p\eta} &\approx \frac{2\lambda^2 \log(p\eta/\mu)}{3(2\pi)^{2(D-1)}} \int_{\infty}^1 dv v^{D-2} \int d^{D-1}l_1 d^{D-1}l_2 d^{D-1}l_3 \delta^{(D-1)}(\vec{l}_1 + \vec{l}_2 + \vec{l}_3) \times \\ &\times h^*(l_1)h(l_1v)h^*(l_2)h(l_2v)h^*(l_3)h(l_3v) A_+A_- [v^{i\mu} + v^{-i\mu}], \quad l_{1,2,3} = |\vec{l}_{1,2,3}|. \end{aligned} \quad (5)$$

[†]The justification of all the listed here approximations can be found in [1], [2], [3], [9], [15], [10].

The lower limit of integration over η_3 we cut by μ , because at $p\eta \gg \mu$ the $d^{D-1}l_i$, $d\eta_3$ and dv integrals are rapidly convergent due to the oscillations of $h(x)$, while we care only about the leading IR contribution. It is worth stressing here that such a convergence of the dv and $d^{D-1}l_i$ integrals is true in the sense of the generalized functions. The latter fact is related to the discussed below behavior of the modes with high momenta.

Note that $n_p(\eta)$ and $k_p(\eta)$ are functions of the physical momentum, $p\eta$, rather than separately depend on the momentum, p , and on the time, η . That is natural because of the spatial homogeneity and the invariance of the PP dS metric under the simultaneous rescaling $\vec{x} \rightarrow \sigma\vec{x}$, $\eta \rightarrow \sigma\eta$.

It was explained in [3], [9], [10], [15] that such large log contributions in D^K appear due to the particle creation in dS space (see also the discussion below). In the expanding PP the creation of particles with comoving momentum p effectively starts after some moment of time $\eta_* \lesssim \mu/p$, because the modes with high momenta, $p\eta \gg \mu$, do not feel the curvature of the space-time and behave as if they are in flat space. As the result n_p and k_p are proportional to the proper time, $\log(\mu/p\eta)$, elapsed from μ/p to η . The coefficient of the proportionality is just the particle production rate. It is worth stressing here that the presence of the large k_p , which is comparable to n_p , signals that there is the strong backreaction on the initial state $|vac\rangle$ [3],[10],[15].

For the further reference it is instructive to study also the IR behavior of n_p and k_p for the out-Jost-harmonics. Although UV behavior of the correlation functions for the out-Jost-harmonics is different from the proper flat space type, we are interested here in the IR limit, where out-Jost-harmonics may play a crucial role[‡]. In fact, in condensed matter physics it is the frequent situation that in the IR limit one has to perform a Bogolyubov rotation to some harmonics whose UV properties may be different from the proper ones, however they properly describe the IR physics. The seminal example is the BCS theory for superconductivity.

In particular, we will see that the proper solution of the IR limit of the system of Dyson-Schwinger (DS) equations is obtained via the out-Jost-harmonics. The hint for that comes from the following observation. For the out-Jost-harmonics the leading IR two-loop contribution has a crucial difference wrt that of BD harmonics or any other α -harmonics. In fact, the out-Jost-harmonics behave as $h(p\eta_{3,4}) \approx A(p\eta_{3,4})^{i\mu}$ in the future infinity, $p\eta_{3,4} \rightarrow 0$, where A is some mass dependent complex constant. Then it is straightforward to show that n_p has the same form as (5) with $|A|^2 v^{i\mu}$ instead of $[|A_+|^2 v^{i\mu} + |A_-|^2 v^{-i\mu}]$. At the same time k_p does not receive any large contributions in the IR limit in question, i.e. it is negligible in comparison with n_p . This is going to be an important observation for the derivation of the kinetic equation below.

It is worth stressing here that for the out-Jost-harmonics the dv and $d^{D-1}l_i$ integrals in the obtained expressions for n_p and k_p are also convergent. In fact, the situation in the ϕ^4 theory is even simpler than in the ϕ^3 case [3].

3.2 Complementary series

We continue with the consideration of the complementary series, $\frac{D-1}{2} > m$, corresponding to the imaginary μ , i.e. to the real index of the solution of the Bessel equation $h(p\eta)$. Below

[‡]While the theory under consideration shows the proper UV behavior in the BD state, it does not do that in any other α -state. The reason for that is as follows. While BD harmonics behave as single waves $e^{ip\eta}$ in the UV limit, $p\eta \rightarrow \infty$, the other α -harmonics behave as linear combinations of $e^{ip\eta}$ and $e^{-ip\eta}$ in the same limit.

we use the notation $\nu = -i\mu$. Then for the in-harmonics $h = J_\nu + iY_\nu$, where both Bessel functions J_ν and Y_ν are real in the case of the real index ν . Expanding them near zero, we get $Y_\nu(x) \approx A_- x^{-\nu} + Bx^{-\nu+2}$ and $J_\nu(x) \approx A_+ x^\nu$. Because of the possible differences between the behavior of h and h^* near zero we have to pay attention separately to k_p and k_p^* .

The contributions to n_p and k_p , k_p^* can be expressed as

$$\begin{aligned} n_p(\eta) &\approx -\frac{2\lambda^2}{3(2\pi)^{2(D-1)}} \int_{-\infty}^0 \frac{du}{u} \int_{-\infty}^0 \frac{dv}{v^{2D-1}} F[v] h(uv) h^*\left(\frac{u}{v}\right) \theta[uv - p\eta] \theta\left[\frac{u}{v} - p\eta\right], \\ k_p(\eta) &\approx \frac{4\lambda^2}{3(2\pi)^{2(D-1)}} \int_{-\infty}^0 \frac{du}{u} \int_{-\infty}^0 \frac{dv}{v^{2D-1}} F[v] h^*(uv) h^*\left(\frac{u}{v}\right) \theta[uv - p\eta] \theta\left[\frac{u}{v} - uv\right], \\ k_p^*(\eta) &\approx \frac{4\lambda^2}{3(2\pi)^{2(D-1)}} \int_{-\infty}^0 \frac{du}{u} \int_{-\infty}^0 \frac{dv}{v^{2D-1}} F[v] h(uv) h\left(\frac{u}{v}\right) \theta[uv - p\eta] \theta\left[\frac{u}{v} - uv\right], \end{aligned} \quad (6)$$

with the use of the following notations

$$F(\eta_3, \eta_4) = \int \left[\prod_{i=1}^3 d^{D-1} q_i h(q_i \eta_3) h^*(q_i \eta_4) \right] (\eta_3 \eta_4)^{D-2} \delta^{(D-1)}(\vec{p} - \vec{q}_1 - \vec{q}_2 - \vec{q}_3), \quad (7)$$

which, after the change of integration variables $u = p\sqrt{\eta_3 \eta_4}$, $v = \sqrt{\frac{\eta_3}{\eta_4}}$, can be expressed as

$F(\eta_3, \eta_4) = \frac{p^2}{(uv)^{2(D-1)}} u^{2(D-2)} F[v]$, where $F[v] = F^*[1/v]$ is some function of one variable v .

The leading correction to n_p and k_p is given by (6) where from the Hankel functions, $h(uv)$ and $h(u/v)$, we single out only Y 's. Such a contribution gives for n_p and k_p the inverse power like behavior in $p\eta$, which, however, cancels out after the substitution into D^K , because Y is real. The next order is obtained as follows. One also has to express $h(p\eta_{1,2})$ through J_ν and Y_ν in the full propagator D^K . Then from one of the four h 's ($h(p\eta_{1,2})$ and $h(uv)$, $h(u/v)$) we have to single out J_ν , while from the other three — Y_ν 's. This expression doesn't cancel out and provides the leading IR contribution to D^K :

$$\begin{aligned} D^K(\eta_1, \eta_2, p) &= \frac{8\lambda^2 A_-^3 A_+ \eta^{D-1}}{3(2\pi)^{2(D-1)}} \int_{-\infty}^0 du u^{-1-2\nu} \int_{-\infty}^0 dv v^{1-2D} F[v] \\ &\left\{ \left[-1 + \left(\frac{u}{p\eta v} \right)^{2\nu} \right] \theta\left[uv - \frac{u}{v}\right] \theta[-p\eta + uv] + \left[1 - \left(\frac{uv}{p\eta} \right)^{2\nu} \right] \theta\left[-uv + \frac{u}{v}\right] \theta\left[-p\eta + \frac{u}{v}\right] \right\}. \end{aligned} \quad (8)$$

After the straightforward manipulations the obtained expression can be reduced to:

$$\begin{aligned} D^K(\eta_1, \eta_2, p) &= -\frac{8\lambda^2 A_-^3 A_+ \eta^{D-1} \log(p\eta)}{3(2\pi)^{2(D-1)} (p\eta)^{2\nu}} \\ &\left\{ \int_1^\infty dv v^{1-2D} F[v] \left[-\frac{1}{2\nu} v^{2\nu} + \left(\frac{1}{v} \right)^{2\nu} \right] - \int_0^1 dv v^{1-2D} F[v] \left[\frac{1}{2\nu} (v)^{-2\nu} + v^{2\nu} \right] \right\}. \end{aligned} \quad (9)$$

The integral over v is convergent in the IR limit (as $v \rightarrow \infty$) if $D > 1 + 4\nu$. In the UV limit ($v \rightarrow 0$) it is convergent in the sense of the generalized function.

For the out-harmonics the situation is a bit different. In this case $h = J_\nu$, $h^* = Y_\nu$. The straightforward calculation shows that

$$\begin{aligned} n_p(\eta) &\propto \lambda^2 A_- A_+ \log(p\eta) \int_0^1 dv F[v] v^{2\nu+1-2D}, \\ k_p(\eta) &\propto \lambda^2 A_-^2 (p\eta)^{-2\nu} \int_1^0 dv F[v] v^{2\nu+1-2D}, \\ k_p^*(\eta) &\propto \lambda^2 A_+^2 \mu^{2\nu} \int_0^1 dv F[v] v^{-2\nu+1-2D} + O((p\eta)^{2\nu}). \end{aligned} \quad (10)$$

After the substitution into the Keldysh propagator the leading contribution comes from k_p^* and is as follows:

$$D^K \approx \frac{\lambda^2 A_-^2 A_+^2 \mu^{2\nu}}{3\nu (2\pi)^{D-1} (p\eta)^{2\nu}} \eta^{D-1} \int_0^1 dv v^{-2\nu+1-2D} F[v] \quad (11)$$

Thus, for the complementary series the loop contributions are power like, i.e. stronger than for the case of the principal series.

Because of the character of these IR contributions for the light fields we do not yet understand their physical meaning and think that the obtained in the next section kinetic equation is not applicable for the fields from the complementary series. In the case of the complementary series we do not yet know how to perform the summation of the leading IR contributions from all loops. But on general physical grounds and from the two loop result we expect that complementary series will show stronger IR effects than the heavy fields from the principal series, which is under study below.

4 Kinetic equation

Although λ^2 is small the product $\lambda^2 \log(\mu/p\eta)$ can become large as $p\eta \rightarrow 0$. Hence, higher loops are not suppressed in comparison with the tree-level contribution. Then, one has to perform the summation of the leading IR contributions from all loops. This can be achieved via an approximate IR solution of the system of DS equations in the non-stationary diagrammatic technique. We can do that only for the case of the principal series, $m > (D-1)/2$, at least because in this case harmonics oscillate in the IR limit. As the result there is a clear separation of scales between the time dependence of the harmonics $h(p\eta)$ and that of $n_{p\eta}$ and $k_{p\eta}$. This allows to simplify DS equations in the limit under consideration and eventually to solve them approximately. In the case of the complementary series, however, the character of the IR contributions to D^K and to the vertex does not allow to have a clear kinetic interpretation.

Putting it in other words, to move further it is worth observing that above we have calculated the loop corrections to the Keldysh propagator under the assumption that n_p and k_p retain their initial values throughout all the time evolution. In fact, in the calculations of the previous section we have used tree-level propagators, i.e. their initial values. To make the problem

self-consistent one has to take into account the change of n_p and k_p in time. As we will see, this also allows to reduce the problem to the solution of the DS equations for the non-stationary diagrammatic technique. The latter ones represent a system of equations for the matrix propagators, self-energy and for the vertex. In some circumstances, as we will see, this system can be simplified and reduced to a single equation.

We assume that the evolution had started with some density perturbation over the BD state at the past infinity of PP. Then the UV behavior of the theory in question is the same as in flat space, but dS invariance is slightly broken. We would like to trace the destiny of these density perturbations in the future infinity, i.e. would like to see whether the theory relaxes back to the dS invariant state or this density explodes causing the modification of the background geometry? The answer to the latter question also can be obtained only after the solution of the IR limit of the non-stationary DS equation.

Thus, we would like to sum up leading contributions, which are powers of $\lambda^2 \log(p\sqrt{\eta_1\eta_2}/\mu)$, and drop the subleading terms, which are suppressed by higher powers of λ and/or powers of $\lambda^2 \log(\eta_1/\eta_2)$. Having in mind that retarded and advanced propagators and the vertex do not receive any large contributions in the limit in question[§], we can assume that they take their classical (UV renormalized) values. At the same time we assume that the ansatz for the exact Keldysh propagator is given by (2),(3) with undefined $n_p(\eta)$ and $k_p(\eta)$, where $\eta = \sqrt{\eta_1\eta_2}$.

Then the relevant part of the system of DS equations has the form

$$\begin{aligned}
D^K(\eta_1, \eta_2, p) = & D_0^K(\eta_1, \eta_2, p) - \\
& - \frac{\lambda^2}{6(2\pi)^{2(D-1)}} \int d^{D-1}q_1 d^{D-1}q_2 d^{D-1}q_3 \iint_{\infty}^0 \frac{d\eta_3 d\eta_4}{(\eta_3\eta_4)^D} \delta^{(D-1)}(\vec{p} - \vec{q}_1 - \vec{q}_2 - \vec{q}_3) \\
& \left[3 D_0^K(\eta_1, \eta_3, p) D^K(\eta_3, \eta_4, q_1) D^K(\eta_3, \eta_4, q_2) D_0^A(\eta_3, \eta_4, q_3) D_0^A(\eta_4, \eta_2, p) \right. \\
& - \frac{1}{4} D_0^K(\eta_1, \eta_3, p) D_0^A(\eta_3, \eta_4, q_1) D_0^A(\eta_3, \eta_4, q_2) D_0^A(\eta_3, \eta_4, q_3) D_0^A(\eta_4, \eta_2, p) \\
& - \frac{3}{4} D_0^R(\eta_1, \eta_3, p) D^K(\eta_3, \eta_4, q_1) D_0^A(\eta_3, \eta_4, q_2) D_0^A(\eta_3, \eta_4, q_3) D_0^A(\eta_4, \eta_2, p) \\
& + D_0^R(\eta_1, \eta_3, p) D^K(\eta_3, \eta_4, q_1) D^K(\eta_3, \eta_4, q_2) D^K(\eta_3, \eta_4, q_3) D_0^A(\eta_4, \eta_2, p) \\
& - \frac{3}{4} D_0^R(\eta_1, \eta_3, p) D^K(\eta_3, \eta_4, q_1) D_0^R(\eta_3, \eta_4, q_2) D_0^R(\eta_3, \eta_4, q_3) D_0^A(\eta_4, \eta_2, p) \\
& - \frac{1}{4} D_0^R(\eta_1, \eta_3, p) D_0^R(\eta_3, \eta_4, q_1) D_0^R(\eta_3, \eta_4, q_2) D_0^R(\eta_3, \eta_4, q_3) D^K(p\eta_4, p\eta_2) \\
& \left. + 3 D_0^R(\eta_1, \eta_3, p) D_0^R(\eta_3, \eta_4, q_1) D^K(\eta_3, \eta_4, q_2) D^K(\eta_3, \eta_4, q_3) D^K(\eta_4, \eta_2, p) \right] \quad (12)
\end{aligned}$$

where $p = |\vec{p}|$, $q_{1,2,3} = |\vec{q}_{1,2,3}|$ and $D_0^{A,R,K}$ are the Fourier transforms of the initial values of the retarded, advanced and Keldysh propagators[¶], D^K — is the exact Keldysh propagator.

[§]Of course there are large IR contributions to $D^{A,R}$ and to the vertex, which are coming from those in D^K . Correspondingly they are suppressed by the higher powers of λ , because they appear in the higher loops. Actually all one-loop contributions to the vertex have similar structure to the one shown in (4) with one momentum integration less. It is straightforward to see that because of that there is no any large IR contribution to the vertex at the λ^2 order.

[¶]I.e. D_0^K is also given by (2),(3) with some initial values $n_p^{(0)}$ and $k_p^{(0)}$.

This equation is covariant under the Bogolyubov rotations between different harmonics, $h(x)$. Because we are interested in its solution in the IR limit we do not have to care about the proper UV behavior and have to check the situation for all possible α -harmonics.

We would like to pick out the largest IR contribution from the integral on the RHS of (12). The calculation is just a straightforward generalization of the above two-loop one. Similarly to [3], [10], [15], for all α -harmonics, including BD ones, we obtain that the ansatz (2), (3) solves DS equation in the IR limit in question. But k_p is comparable to n_p . This means that the backreaction on the background state (specified by the choice of the harmonics) is big. The only exception is given by the out-Jost-harmonics. For these harmonics k_p remains zero if its initial value was zero. Moreover, small perturbations of k_p relax back to zero. For the ϕ^3 theory this was shown in [15]. For the ϕ^4 theory the situation is similar.

As the result, with the use of the out-Jost-harmonics the IR limit of the DS equation is solved by (2), (3) with $k_p = 0$. We would like to convert the integral DS equation (12) into the integrodifferential form, i.e. into the form of the kinetic equation [24]. This is done via a kind of the renormalization group procedure as follows [15]. In the given settings $n_p^{(0)}$ is the particle density at some moment after $\eta_* \sim \mu/p$. In fact, as we have mentioned above and explain in the next section, before this moment all the kinetic processes, except may be the irrelevant scattering one, are suppressed. Hence, $n_p(\eta)$ remains constant and is equal to $n_p^{(0)}$. Then, from (12) it is straightforward to derive that the difference between $n_p(\eta)$ and $n_p^{(0)} = n_p(\eta_*)$ is proportional to the proper time elapsed from η_* to η . The coefficient of the proportionality is the collision integral — the RHS of the kinetic equation:

$$\begin{aligned} \frac{n_p(\eta) - n_p(\eta_*)}{\log(\eta) - \log(\eta_*)} \rightarrow \frac{dn_{p\eta}}{d\log(p\eta)} = & -\frac{\lambda^2 |A|^2}{6} \int \frac{d^{D-1}l_1}{(2\pi)^{D-1}} \frac{d^{D-1}l_2}{(2\pi)^{D-1}} \int_{\infty}^0 dv v^{D-2} \\ & \left\{ 3\text{Re} \left[v^{i\mu} \ h^*(l_1)h^*(l_2)h \left(\left| \vec{l}_1 + \vec{l}_2 \right| \right) h(l_1v)h(l_2v)h^* \left(\left| \vec{l}_1 + \vec{l}_2 \right| v \right) \right] \times \right. \\ & \times \left[(1 + n_{p\eta})n_{l_1}n_{l_2}(1 + n_{|\vec{l}_1 + \vec{l}_2|}) - n_{p\eta}(1 + n_{l_1})(1 + n_{l_2})n_{|\vec{l}_1 + \vec{l}_2|} \right] \\ & + 3\text{Re} \left[v^{i\mu} \ h^*(l_1)h(l_2)h \left(\left| \vec{l}_1 - \vec{l}_2 \right| \right) h(l_1v)h^*(l_2v)h^* \left(\left| \vec{l}_1 - \vec{l}_2 \right| v \right) \right] \times \\ & \times \left[(1 + n_{p\eta})n_{l_1}(1 + n_{l_2})(1 + n_{|\vec{l}_1 - \vec{l}_2|}) - n_{p\eta}(1 + n_{l_1})n_{l_2}n_{|\vec{l}_1 - \vec{l}_2|} \right] \\ & + \text{Re} \left[v^{i\mu} \ h^*(l_1)h^*(l_2)h^* \left(\left| \vec{l}_1 + \vec{l}_2 \right| \right) h(l_1v)h(l_2v)h \left(\left| \vec{l}_1 + \vec{l}_2 \right| v \right) \right] \times \\ & \times \left[(1 + n_{p\eta})n_{l_1}n_{l_2}n_{|\vec{l}_1 + \vec{l}_2|} - n_{p\eta}(1 + n_{l_1})(1 + n_{l_2})(1 + n_{|\vec{l}_1 + \vec{l}_2|}) \right] \\ & + \text{Re} \left[v^{i\mu} \ h(l_1)h(l_2)h \left(\left| \vec{l}_1 + \vec{l}_2 \right| \right) h^*(l_1v)h^*(l_2v)h^* \left(\left| \vec{l}_1 + \vec{l}_2 \right| v \right) \right] \times \\ & \times \left[(1 + n_{p\eta})(1 + n_{l_1})(1 + n_{l_2})(1 + n_{|\vec{l}_1 + \vec{l}_2|}) - n_{p\eta}n_{l_1}n_{l_2}n_{|\vec{l}_1 + \vec{l}_2|} \right] \Big\}. \end{aligned} \quad (13)$$

In the process of the derivation of this equation we have neglected p in comparison with $q_{1,2,3}$ on its RHS, denoted $\vec{l}_{1,2,3} = \vec{q}_{1,2,3}\eta$ and assumed that on the RHS of (12) $n_{k\eta}$ is much slower function of time than $h(k\eta)$. The latter fact is true because of the above mentioned separation of scales. As the result we can safely take out all n 's from the argument of the time integral on the RHS of (12) and substitute D_0^K by D^K .

Note that one can reproduce n_p in eq.(5) from (13) if he will use Hankel functions in place of $h(x)$, change $|A|^2 v^{i\mu}$ to $[|A_+|^2 v^{i\mu} + |A_-|^2 v^{-i\mu}]$ and put all n 's to zero on the RHS of (13).

5 Solution of the kinetic equation

The kinetic equation (13) does not possess Planckian distribution as its solution, because of the violation of energy conservation in the dS background gravitational field. It can be mapped to the kinetic equation in flat space via the substitution of the harmonics $\eta^{(D-1)/2} h(k\eta)$ by the plane waves $e^{i\epsilon(k)t}$ in the collision integral. In the latter case one has δ -functions ensuring energy conservation on the RHS of (13) instead of the integrals of h 's. That in particular means that for high energy modes, $k\eta \gg \mu$, the kinetics in dS space is the same as in flat one: they can scatter off each other, but cannot be created in various processes involving dS background.

Thus, suppose we have started at past infinity of PP with some very mild density perturbation over the BD state. After the Bogolyubov rotation to the out-Jost-harmonics, one has some initial values of n_p and k_p . As can be understood from the discussion in the previous paragraph, for the given p the density n_p and the anomalous average k_p practically do not change before the moment $\eta_* \sim \mu/p$. After this moment they begin to evolve according to the coupled system of kinetic equations for n_p and k_p . (To simplify the presentation we do not show them here. Similar equations for the ϕ^3 theory can be found in [3], [10].) If k_p is sufficiently small it relaxes to zero [15] and the problem is reduced to the solution of (13).

Now if the initial value of n_p after the rotation to the out-Jost-harmonics is much smaller than 1 we can use the following approximations:

$$\begin{aligned}
(1 + n_{p\eta})n_{l_1}n_{l_2}(1 + n_{|\vec{l}_1 + \vec{l}_2|}) - n_{p\eta}(1 + n_{l_1})(1 + n_{l_2})n_{|\vec{l}_1 + \vec{l}_2|} &\approx 0 \\
(1 + n_{p\eta})n_{l_1}(1 + n_{l_2})(1 + n_{|\vec{l}_1 - \vec{l}_2|}) - n_{p\eta}(1 + n_{l_1})n_{l_2}n_{|\vec{l}_1 - \vec{l}_2|} &\approx n_{l_1} \\
(1 + n_{p\eta})n_{l_1}n_{l_2}n_{|\vec{l}_1 + \vec{l}_2|} - n_{p\eta}(1 + n_{l_1})(1 + n_{l_2})(1 + n_{|\vec{l}_1 + \vec{l}_2|}) &\approx -n_{p\eta} \\
(1 + n_{p\eta})(1 + n_{l_1})(1 + n_{l_2})(1 + n_{|\vec{l}_1 + \vec{l}_2|}) - n_{p\eta}n_{l_1}n_{l_2}n_{|\vec{l}_1 + \vec{l}_2|} &\approx 1.
\end{aligned} \tag{14}$$

Because of the rapid oscillations of $h(x)$ as $x \rightarrow \infty$ the integrals on the RHS of (13) are saturated at $l_i \sim \mu$. At the same time $p\eta \ll \mu$. Furthermore it is natural to assume that $n_{l_i \sim 1} \ll n_{p\eta}$ in the situation of the very small initial density perturbation and the vanishing production of the high momentum modes. Hence, we can neglect the second term on the RHS of (13) in comparison with the third and the fourth ones.

As the result (13) is reduced to:

$$\begin{aligned}
\frac{dn_{p\eta}}{d \log(p\eta)} &\approx \Gamma_1 n_{p\eta} - \Gamma_2, \quad \text{where} \\
\Gamma_1 &= \frac{\lambda^2 |A|^2}{6} \int \frac{d^{D-1}l_1}{(2\pi)^{D-1}} \frac{d^{D-1}l_2}{(2\pi)^{D-1}} \int_{-\infty}^0 dv v^{D-2} \times \\
&\times \text{Re} \left[v^{i\mu} h^*(l_1)h^*(l_2)h^* \left(\left| \vec{l}_1 + \vec{l}_2 \right| \right) h(l_1 v)h(l_2 v)h \left(\left| \vec{l}_1 + \vec{l}_2 \right| v \right) \right], \\
\Gamma_2 &= \frac{\lambda^2 |A|^2}{6} \int \frac{d^{D-1}l_1}{(2\pi)^{D-1}} \frac{d^{D-1}l_2}{(2\pi)^{D-1}} \int_{-\infty}^0 dv v^{D-2} \times \\
&\times \text{Re} \left[v^{i\mu} h(l_1)h(l_2)h \left(\left| \vec{l}_1 + \vec{l}_2 \right| \right) h^*(l_1 v)h^*(l_2 v)h^* \left(\left| \vec{l}_1 + \vec{l}_2 \right| v \right) \right].
\end{aligned} \tag{15}$$

Here Γ_1 and Γ_2 are the particle decay and production rates, correspondingly. Note that $p\eta$ is reducing to zero in the approach towards the future infinity.

The obtained equation (15) has the solution with the flat stationary point distribution $n_{p\eta} = \Gamma_2/\Gamma_1$, which corresponds to the situation when the production (gain) of particles on the level $p\eta$ is equilibrated by the particle decay (loss) from the same level. Note that here we are talking about the number density per comoving volume, which would stay constant if there were no particle decay and production processes.

The obtained solution is self-consistent for the large enough μ , because then $\Gamma_2/\Gamma_1 \approx e^{-3\pi\mu} \ll 1$. Note that the equilibrium distribution is $n \approx e^{-3\pi\mu}$ and is not quite Gibbons-Hawking one. Apparently, it looks like the thermal Boltzmann one, but the temperature depends on the power of the selfinteraction potential. In fact, the stationary distribution in the ϕ^3 theory is $n \approx e^{-2\pi\mu}$.

So the result of the summation of the large IR contributions may lead to the finite exact two point functions. But what if the evolution had started with some quite strong density perturbation (which is, however, still smaller than the cosmological constant) over the BD vacuum state? Now we are going to show that there is another very peculiar solution of the kinetic equation under consideration. See [3], [10] for the similar discussion in the case of ϕ^3 theory.

Suppose that due to the particle creation by the background gravitational field and by the particle decays from the other levels the density per comoving volume on the given level with $p\eta \ll \mu$ became big in comparison with 1. Taking into account the flatness of the spectrum in dS space, it is natural to expect that for the harmonics with the low physical momenta the density very slowly depends on its argument. Hence, we can assume that $n(p\eta) \approx n(q_{1,2,3}\eta)$ for $p\eta \ll \mu$ and $q_{1,2,3}\eta \ll \mu$.

Then, we can make the following approximations:

$$\begin{aligned}
(1 + n_{p\eta})n_{l_1}n_{l_2}(1 + n_{l_3}) - n_{p\eta}(1 + n_{l_1})(1 + n_{l_2})n_{l_3} &\approx 0 \\
(1 + n_{p\eta})n_{l_1}(1 + n_{l_2})(1 + n_{l_3}) - n_{p\eta}(1 + n_{l_1})n_{l_2}n_{l_3} &\approx 2n_{p\eta}^3 \\
(1 + n_{p\eta})n_{l_1}n_{l_2}n_{l_3} - n_{p\eta}(1 + n_{l_1})(1 + n_{l_2})(1 + n_{l_3}) &\approx -2n_{p\eta}^3 \\
(1 + n_{p\eta})(1 + n_{l_1})(1 + n_{l_2})(1 + n_{l_3}) - n_{p\eta}n_{l_1}n_{l_2}n_{l_3} &\approx 4n_{p\eta}^3
\end{aligned} \tag{16}$$

and accept that on the RHS of (13) the main contribution to the l_i integrals comes from the region where $l_i \ll \mu$, because $n(x) \gg n(y)$ if $x \ll \mu$ and $y \gg \mu$. As the result the kinetic equation reduces to:

$$\begin{aligned}
& \frac{dn_{p\eta}}{d\log(p\eta)} \approx -\bar{\Gamma} n_{p\eta}^3, \quad \text{where} \\
\bar{\Gamma} = & \frac{\lambda^2 |A|^2}{3} \int^{|l_1| < \mu} \frac{d^{D-1} l_1}{(2\pi)^{D-1}} \int^{|l_2| < \mu} \frac{d^{D-1} l_2}{(2\pi)^{D-1}} \int_{-\infty}^0 dv v^{D-2} \\
& \left\{ 3 |A|^2 A \text{Re} \left[\left(\frac{v l_2 |\vec{l}_1 - \vec{l}_2|}{l_1} \right)^{i\mu} h(l_1 v) h^*(l_2 v) h^* \left(|\vec{l}_1 - \vec{l}_2| v \right) \right] \right. \\
& - (A^*)^3 \text{Re} \left[\left(\frac{v}{l_1 l_2 |\vec{l}_1 + \vec{l}_2|} \right)^{i\mu} h(l_1 v) h(l_2 v) h \left(|\vec{l}_1 + \vec{l}_2| v \right) \right] \\
& \left. + 2 A^3 \text{Re} \left[\left(v l_1 l_2 |\vec{l}_1 + \vec{l}_2| \right)^{i\mu} h^*(l_1 v) h^*(l_2 v) h^* \left(|\vec{l}_1 + \vec{l}_2| v \right) \right] \right\}. \tag{17}
\end{aligned}$$

Note that $\bar{\Gamma}$ is independent of p . This equation has the solution:

$$n_{p\eta} \approx \frac{1}{\sqrt{2 \bar{\Gamma} \log(\eta/\eta_*)}}, \tag{18}$$

where $\eta_* = \frac{\mu}{p} e^{-\frac{C}{2\bar{\Gamma}}}$ and C is the integration constant, which depends on the initial conditions. The obtained solution is valid if $\mu/p = \eta_* > \eta > \eta_*$.

Thus, we see that there is a singular solution of the kinetic equation under consideration, which corresponds to the explosion of the particle number density per comoving volume within a finite proper time. Of course such an explosion wins against the expansion of the PP, because $D^K \propto \eta^{D-1}/\sqrt{\log \eta/\eta_*}$. Hence, the energy-momentum tensor of the created particles becomes huge and the backreaction has to be taken into account. As the result the dS space gets modified. But that is the problem for a separate study. At this point we just would like to stress that we see catastrophic IR effects even for the massive fields. It is natural to expect that for the light fields the situation will be even more dramatic.

6 Comments on the contracting PP and global dS (instead of Conclusions)

Contracting PP of dS space is interesting at least because it is complementary to the expanding PP within global dS space and we find it quite dangerous to study such a geodesically incomplete subspace as PP alone. Contracting PP is represented by the same metric, $ds^2 = \frac{1}{\eta} [d\eta^2 - d\vec{x}^2]$, as the expanding one, but now the conformal time η flows in the proper direction — from zero, at the past, to infinity, in the future.

From (13) one can straightforwardly obtain the kinetic equation in the contracting PP, if he will consider perfectly spatially homogeneous states. The latter situation while being stable in the expanding PP, is unstable in the contracting one under small inhomogeneous

density perturbations. However, it is still instructive to consider such an ideal situation in the contracting PP^{||}. In this section we restrict ourselves to the case of the principal series.

To perform the map between the expanding and contracting PPs both in the few loop calculations and in the kinetic equation one just has to flip the limits of $d\eta$ integrations and change $h(x)$ and $h^*(x)$, because of the exchange between positive and negative energy states under the flip of time.

Then, it is straightforward to see that loop corrections to D^K have the explicit IR divergence because particle creation starts right at the moment η_0 , when we switch on selfinteractions. The divergence reveals itself via the impossibility to move the η_0 to the past infinity. Due to the blue shifting of all modes, however, all the relevant kinetic processes stop after $\eta_* \sim \mu/p$. As the result the two loop divergence in question is proportional to $\log(\eta/\eta_0)$, when $p\eta \ll \mu$ and to $\log(\mu/p\eta_0)$, when $p\eta \gg \mu$. The prefactors are easily derivable, in view of the above discussion. Also it is straightforward to show that for the in-Jost-harmonics (Bessel functions in place of h 's) of the contracting PP the k_p behaves similarly to that of the out-Jost-harmonics of the expanding PP.

In conclusion, in the contracting PP we can find similar solutions of the kinetic equation to those, which have been found in the previous section. E.g. eq. (15) is mapped to

$$\frac{dn(\eta)}{d\log(\eta/\eta_0)} \approx -\Gamma_1 n(\eta) + \Gamma_2, \quad (19)$$

which shows a very peculiar phenomenon that despite that in two loops η_0 — the moment of switching on selfinteractions — cannot be taken to the past infinity, after the summation of all loops we may find the theory at the stationary point state, $n_p = \Gamma_2/\Gamma_1$, which allows one to remove the IR cutoff, η_0 .

At the same time the solution (18) is mapped to:

$$n_p(\eta) \approx \frac{1}{\sqrt{2\Gamma \log(\eta_*/\eta)}}, \quad (20)$$

where $\eta_* = \eta_0 e^{C/2\bar{\Gamma}} < \mu/p$. Of course whether the field theory state goes into (20) or to (19) depends on the initial conditions.

The situation in the global dS space is even more interesting. On the one hand all sorts of contributions to the collision integral in global dS space are sums of those in expanding and contracting PPs^{**}. But depending on the choice of harmonics we either have IR divergence in k_p , or k_p is divergent as the system advances towards the future infinity. Or even more generic situation for α -vacua is when k_p has both types of such divergences simultaneously. As the result there is no any choice of harmonics in which k_p is negligible in comparison with n_p , which probably means that there is no any stationary state in global dS and the backreaction on the background geometry is strong with any initial conditions.

^{||}Actually it is not very hard to find the inhomogeneous extension of the kinetic equation (13). In the case when the particle density starts to depend on the spatial position $n_p = n_p(x)$ one has to substitute $\eta d/d\eta$ on the LHS of (13) by $\eta \partial_\eta + \eta^2 \vec{p} \vec{\partial}_x$.

^{**}For the case of the Euclidian vacuum the relative signs of the two contributions coming from the expanding and contracting PPs can be different depending on whether D is even or odd [9]. But that does not conceptually affect our discussion.

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